Cable Tension Optimization

A common problem in the analysis of a cable-stayed bridge is the determination of initial cable tension forces that — in combination with other loading, the construction sequence, and time-dependent material effects — gives the structure its desired final geometry and internal forces. LARSA 4D Bridge Plus provides two solutions to this process. The first determines cable tension forces in a model in which the structure is constructed in a single step. The second is based on the unit-load method and is used for models with a construction sequence.

We call these procedures model optimization. Optimization is the term from mathematics of finding a minimum of a function. These procedures are used in LARSA 4D to minimize deflection.

Iteration Using Final Cable Tension

In a nonlinear structure such as one with cables, one cannot solve directly for the set of cable forces at cable installation that is needed to achieve a chosen deformed state once other loading has been applied. If all of the deformation on the structure takes place after the cable is tensioned, and the goal is to have the base of the cable stay at its undeformed location, then one can make use of the fact that the cable jacking force will match exactly with the final tension in the cable. If there is no change in axial force, the cable has not deformed.

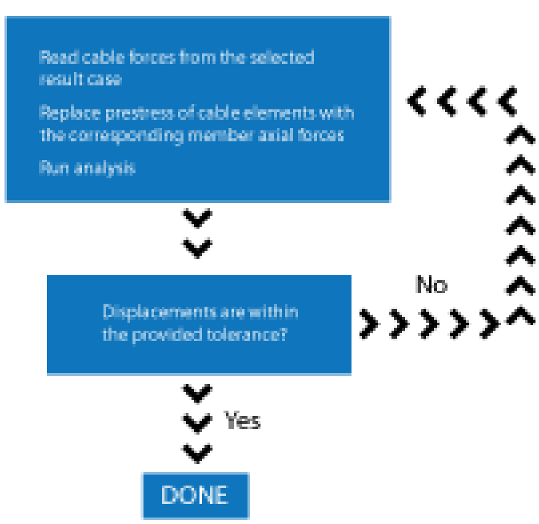

Figure 1. Flow chart of the method to iterate using final cable tension.

A simple procedure that has been carried out by engineers by hand has been to start the cables with minimal jacking force, solve for the axial forces in the cables at the end of construction, take those axial forces and use them as the cable jacking forces the next time around, and then repeat this process until the tensile forces no longer change as a result of other loading on the structure. The procedure is illustrated in Figure 1.

To see why this might work, take the case of a single cable holding up a deck. If the initial cable force is not great enough to hold up the deck, the deck will lower causing the force in the cable to increase. If the cable is jacked too much, the deck will raise and the cable will shorten, causing the cable force to decrease. When the deck is held up in place, the cable does not deform and there is no change in axial force.

We can think of the static analysis as a function $f(x)$, where x is the initial cable force ("pretension" in LARSA 4D) and $f(x)$ is the cable tension after other loading has been applied (the Fx member end force). The goal is to find x such that $f(x) = x$. Actual values for x and f from the first cable of an ordinary cable-stayed bridge are shown in Figure 2. The solution for this cable in isolation is where f intersects the dotted line for $y = x$. Each iterative step moves x from x to $f(x)$.

If there is more than one cable, we can think of f as a function from a vector of cable jacking forces x to a vector of final cable tension values $f(x)$.

This procedure is automated in LARSA 4D. Before we begin:

- The model should be prepared for a Nonlinear Static analysis.
- The model should also be created with joints located at their desired deformed location so that the desired outcome is no displacement under all of the static loading in the model.
- An analysis should be run once to ensure the analysis options are set appropriately.
- An initial guess for cable prestress must be made if it is required for the analysis to complete.
- Then select a result case from the Analysis Results Explorer, the result case with all static loading applied.
- Also note the ID number of a joint whose displacements are

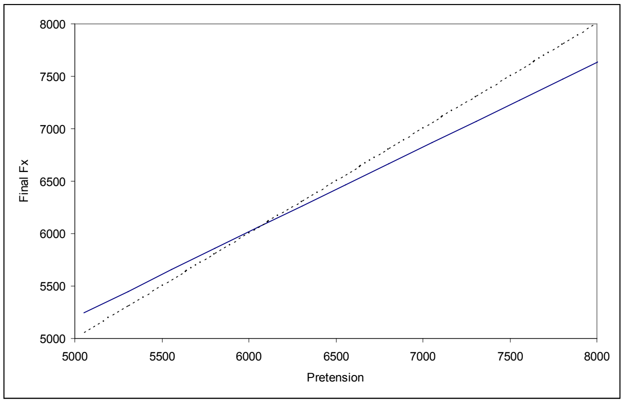

Figure 2. Final cable tension plotted against initial cable prestress force; units are kN.

checked as a stopping criteria.

To start the tool:

- In the LARSA 4D menu go to ***Tools > Geometry Control > Iterative Cable Tension Optimization***. The tool performs the procedure described in this section over all cable elements in the model, in each iteration copying the final axial forces in the selected result case back into the prestress column in the members spreadsheet. The procedure stops when the displacement of the chosen joint reduces to within a tolerance given by the user.

In a typical nine-cable cable-stayed bridge model, we have found that the process requires only roughly five iterations to achieve near-zero displacements. A perfect solution may be impossible due to other structural elements in the model, and in this case the procedure could be left to adjust

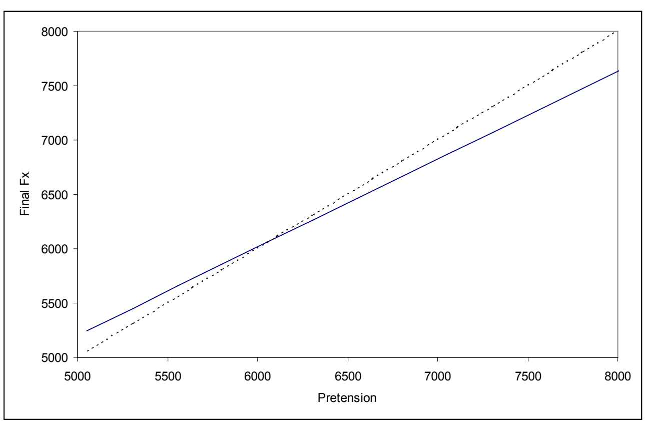

Figure 3. Cable pretension values for nine cables after the first 55 iterations.

pretension values indefinitely. The cable pretension values for the nine cables in a typical model after each iteration are shown in Figure 3. At the first iteration the cables are all set to a common initial prestress force.

Iteration Using Unit Loading and a Flexibility Matrix

In a segmental assembly a cable may be installed after its segment has already deformed due to dead load. The goal here is to achieve zero joint displacement, but because the cable is installed in the middle of the construction sequence the cable is intended to deform as it brings the joint back up to its initial location. Since the cable will deform, the initial cable tension will not match final cable tension, the first procedure is not applicable in this case.

A different method is required in this case. The "unit load method" has been applied in the past to solve this problem. In this method, we apply a unit-tug — i.e. one extra unit of jacking force — to each cable and observe its effect on each of the joints at the bases of the cables (or any other joints on the deck). Then we solve for a factor to apply to the tug to zero-out the displacements at a joint. Take the case of a single cable. At the initial condition, the joint at the base of the cable has displaced by 5 meters. Through a static analysis we determine that adding 1 kN of force to the cable raises the joint by 1 meter. We then conclude that 5 times the 1 kN = 5 kN will raise the joint back to its undeformed location. If the structure has nonlinear behavior 5 kN may not have the effect of 5 times the effect of 1 kN, so the process must be iterated until the displacement comes within tolerable limits.

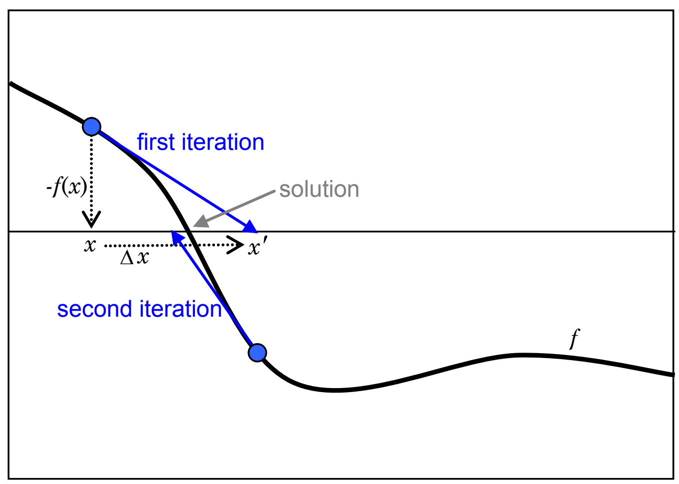

Figure 4. Illustration of Newton-Raphson in one dimension.

This is the application of a procedure used in many other fields of applied mathematics and is a generalization into multiple dimensions of the Newton-Raphson method of finding the solution to $f(x) = 0$. As opposed to the first iterative method described above, $f(x)$ here is a function from a vector of cable pretension forces to a vector of joint deformations in the elevation axis.

The Newton-Raphson method can be summarized as follows: when searching values of x for the one that makes

$f(x) = 0$, a good guess is to use the slope of f to predict where the function is going. This is shown in Figure 4, and formally in Equations 1–2, where f' denotes the derivative of f.

eqn 1. $f'(x) \cdot \Delta x = -f(x)$

eqn 2. $\Delta x = \frac{f(x)}{f'(x)}$

When there is more than one cable this process must be generalized to multiple dimensions, and the iterative step is derived as shown in Equations 3–4. The matrix J, called the Jacobian matrix, represents the slope of the function in each dimension. J_{ij} is the change in displacement at joint i due to a one-unit tug on cable j. Δx is the computed additional initial cable tension that is needed and is added in at the end of the current iteration.

eqn 3. $J \cdot \Delta x = -f(x)$

eqn 4. $\Delta x = -J^{-1} f(x)$

eqn 5. $J = [f(x+u_1)-f(x) \;\; f(x+u_2)-f(x) \;\; f(x+u_3)-f(x) \;\; \cdots]$

J is computed by running a separate static analysis for each column of the matrix (i.e. Equation 5). Each analysis applies a one-unit tug u_i to each cable at the time it is installed, within a Staged Construction Analysis already set up by the user that might additionally contain dead load, time-dependent material effects, and other nonlinear behavior. For each analysis we record the displacement of the joints at the bases of the cables at the end of construction (i.e. at the final construction step) and subtract off the corresponding displacements without the unit tug. Once J is assembled Equation 4 is solved for Δx, and this additional force is added to the initial cable tension for the next iteration of the procedure. On the next iteration J is assembled again, and the process continues until $f(x) = 0$.

If you are familiar with the linear algebra behind finite element analysis, you may recognize Equation 4 as containing a force vector Δx equaling a stiffness matrix J^{-1} multiplied by a displacement vector $f(x)$. J is then a flexibility matrix. However, this optimization procedure works equally well even if J is not a flexibility matrix.

The cable jacking forces can be applied at different times, i.e. at different stages during a construction analysis that takes into account time-dependent material properties, temporary loading, other construction activities, and geometric nonlinearity. The algorithm will find whatever pretension force such that the deformations work out at the end. Or x might not be cable force at all. In another application of optimization to bridge design, take the case of a deck assembled segmentally. Due to camber, each segment must go in in such a way that after the structure deforms the new segment is in its desired location. The segment must start off above its desired location, but by how much? This application calls for optimization of each segment's initial location, in which case J is assembled not through unit-tugs on cables but considering the effect of adding one unit to each joint's initial z-coordinate. The procedure is otherwise carried out the same way.

This optimization method is also available in LARSA 4D Bridge Plus:

- Please note, the method works best when the model is already close to the solution. For this reason, it may be necessary to use the first method on a simplified model to find initial cable tension values before going on.
- As with the first procedure, a Staged Construction Analysis should already be successfully run.
- Create a structure group folder, and then create in this folder a structure group for each separate cable (which might contain multiple cable elements if the cables are broken into pieces).
- Additionally create a structure group containing the joints at the bases of the cables.

To start the tool:

- In the LARSA 4D menu go to ***Tools > Geometry Control > General Model Optimization***.
- Choose Multi-Variable Zero-Finding.
- For the Model Parameter, choose Cable Prestress and select the structure group folder for the cables.
- For the Model Target, choose Zero Displacements at Joints in a Group and choose the structure group for the base joints.
- Choose the result case for which the joint displacements are to be made zero. This is usually the last result case/construction step in the analysis.
- Finally, set a tolerance for the joint displacements, which is the stopping criteria for the iterative process. We have found that on relatively simple models with approximately ten cables around 15 iterations is required.

Figure 5 shows the cable pretension set after each iteration on the nine cables in a time-dependent construction analysis of the same example mentioned earlier. At the first iteration the cables are all set to a common initial prestress force. •

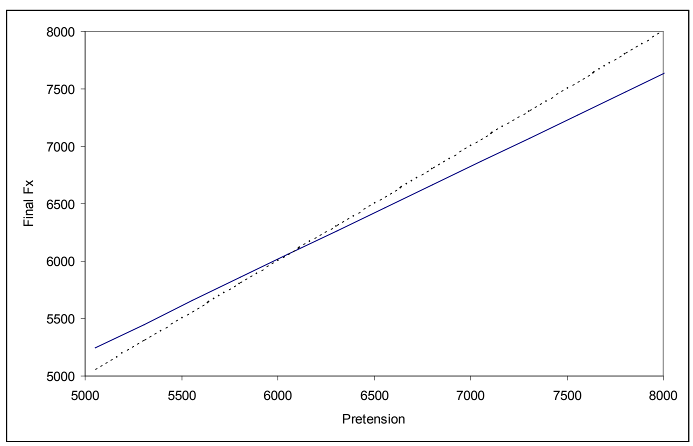

Figure 5. Cable pretension after each iteration using a time-dependent staged construction analysis.